Total Energy of a Flowing Fluid:

The total energy of a simple compressible system consists of three parts: internal, kinetic, and potential energies. On a unit-mass basis, it is expressed as:

$$\boxed{e = u + ke + pe} \quad \Longrightarrow \quad \boxed{e = u + \frac{V^2}{2} + gz} \qquad \text{(kJ/kg)}$$

Where:
V: velocity
Z: the elevation of the system relative to some external reference point

For non-flowing fluid: $e = \overbrace{u}^{\text{Internal energy}} + \underbrace{\frac{V^2}{2}}_{\text{Kinetic energy}} + \overbrace{gz}^{\text{Potential energy}}$

For flowing fluid: $\theta = \overbrace{Pv}^{\text{Flow energy}} + u + \frac{V^2}{2} + gz$

P: Pressure
v: specific volume

The fluid entering or leaving a control volume possesses an additional form of energy (**the flow energy** Pv). Then **the total energy of a flowing fluid on a unit-mass basis** (denoted by θ) becomes:

$$\boxed{\theta = Pv + e} \quad \Longrightarrow \quad \boxed{\theta = Pv + (u + ke + pe)}$$

$$h = Pv + u$$

Then: $$\boxed{\theta = h + ke + pe} \quad \Longrightarrow \quad \boxed{\theta = h + \frac{V^2}{2} + gz} \qquad \text{(kJ/kg)}$$

Where:
h: enthalpy
θ: the total energy of a flowing fluid

Energy Transport by Mass:

Noting that θ is total energy per unit mass, the total energy of a flowing fluid of mass (m) is simply (mθ), provided that the properties of the mass (m) are uniform. Also, when a fluid stream with uniform properties is flowing at a mass flow rate of $\dot{m}$, the rate of energy flow with that stream is:

$$\dot{m}\theta$$

That is:

Amount of energy transport:

$$\boxed{E_{mass} = m\theta = m\left(h + \frac{V^2}{2} + gz\right)} \qquad \text{(kJ)}$$

Rate of energy transport:

$$\boxed{\dot{E}_{mass} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right)} \qquad \text{(KW)}$$

When the kinetic and potential energies of a fluid stream are negligible, as is often the case, these relations simplify to:

$$\boxed{E_{mass} = mh} \qquad \textit{and} \qquad \boxed{\dot{E}_{mass} = \dot{m}h}$$

Energy Analysis of Steady Flow Systems:

During a steady-flow process, the total energy content of a control volume remains constant (E_{CV} =constant), and thus the change in the total energy of the control volume is **zero** ($\Delta E_{CV} = 0$). Therefore, the amount of energy entering a control volume in all forms by (**heat**, **work**, and **mass**) must be equal to the amount of energy leaving it. Then the rate form of the general energy balance reduces for a **steady-flow** process to:

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by (heat, work, and mass)}} = \underbrace{\overset{0\ \text{(steady)}}{dE/dt}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} = 0$$

or *Energy balance*: $\dot{E}_{in} = \dot{E}_{out}$

Rate of net energy transfer in by (heat, work, and mass)

Rate of net energy transfer out by (heat, work, and mass)

Noting that energy can be transferred by heat, work, and mass only, the energy balance in above equation for a general steady-flow system can also be written more explicitly as:

$$\boxed{\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m}\theta = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m}\theta}$$

$$\boxed{\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m}\underbrace{\left(h + \frac{V^2}{2} + gz\right)}_{\text{For each inlet}} = \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m}\underbrace{\left(h + \frac{V^2}{2} + gz\right)}_{\text{For each exit}}}$$

or

$$\boxed{\dot{Q}_1 + \dot{W}_1 + \dot{m}_1\left(h_1 + \frac{V_1^2}{2} + gz_1\right) = \dot{Q}_2 + \dot{W}_2 + \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + gz_2\right)}$$

Dividing above equation by $\dot{m}$ gives the energy balance on a unit-mass basis as:

$$\boxed{q_1 + w_1 + \left(h_1 + \frac{V_1^2}{2} + gz_1\right) = q_2 + w_2 + \left(h_2 + \frac{V_2^2}{2} + gz_2\right)}$$

Where:

$q = \dfrac{\dot{Q}}{\dot{m}}$ and $w = \dfrac{\dot{W}}{\dot{m}}$

The various terms appearing in the above equations are as follows:

$\dot{W}$: **Power.**

$\dot{Q}$: **rate of heat transfer between the control volume and its surroundings.**

When the control volume is losing heat (as in the case of the water heater) → $\dot{Q}$ is negative

If the control volume is well insulated (i.e., adiabatic) → $\dot{Q} = 0$

$\Delta h = h_2 - h_1$

The enthalpy change of a fluid can easily be determined by reading the enthalpy values at the exit and inlet states from the tables.

For ideal gas → $\Delta h = Cp(T_2 - T_1)$

$$\Delta ke = \left(\frac{V_2{}^2 - V_1{}^2}{2}\right)$$

When a fluid stream enters and leaves a steady-flow device at about the same velocity $V_1 = V_2$, the change in the kinetic energy is close to **zero**

$\Delta pe = g(z_2 - z_1)$

The only time the potential energy term is significant is when a process involves pumping a fluid to high elevations and we are interested in the required pumping power.

Some Steady Flow Engineering Devices:

1. Nozzles and Diffusers:

Nozzles and diffusers shown in figure (31) are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses.

A nozzle: is a device that increases the velocity of a fluid at the expense of pressure.

A diffuser: is a device that increases the pressure of a fluid by slowing it down.

That is, nozzles and diffusers perform opposite tasks. The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.

Through **a nozzle** or **a diffuser:**

$\boxed{\dot{Q} \cong 0}$ *and* $\boxed{\dot{W} = 0}$ *and* $\boxed{\Delta pe \cong 0}$ *and* $\boxed{\Delta ke \neq 0}$

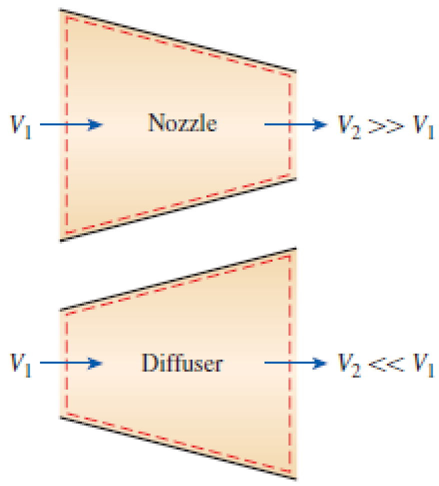


Figure (31) shown nozzles and diffusers

2. Turbines and Compressors:

In steam, gas, or hydroelectric power plants, the device that drives the electric generator is **the turbine**. As the fluid passes through the turbine, work is done against the blades, which are attached to the shaft. As a result, the shaft rotates, and the turbine produces work as shown in figure (32).

Figure (32) Turbine blades attached to the turbine shaft.

Compressors, as well as **pumps** and **fans**, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft. Therefore, compressors involve work inputs. Even though these three devices function similarly, they do differ in the tasks they perform. A fan increases the pressure of a gas slightly and is mainly used to mobilize a gas. A compressor is capable of compressing the gas to very high pressures. Pumps work very much like compressors except that they handle liquids instead of gases.

turbines $\Rightarrow$ ***produce power output*** $\dot{W}_{out}$ and $\dot{Q} \cong 0$ and $\Delta pe \cong 0$

Fans $\Rightarrow$ ***require power input*** $\dot{W}_{in}$ and $\Delta pe \cong 0$

Pumps $\Rightarrow$ ***require power input*** $\dot{W}_{in}$ and $\Delta pe \cong 0$ and $\Delta ke \cong 0$

compressors $\Rightarrow$ ***require power input*** $\dot{W}_{in}$ and $\Delta pe \cong 0$ and $\Delta ke \cong 0$

3. Throttling devices:

Throttling devices are any kind of flow-restricting devices that cause a significant pressure drop in the fluid. Some familiar examples are ordinary adjustable valves, capillary tubes, and porous plugs as shown in figure (33).

Throttling devices produce a pressure drop without involving any work. The pressure drop in the fluid is often accompanied by a large drop in temperature, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.

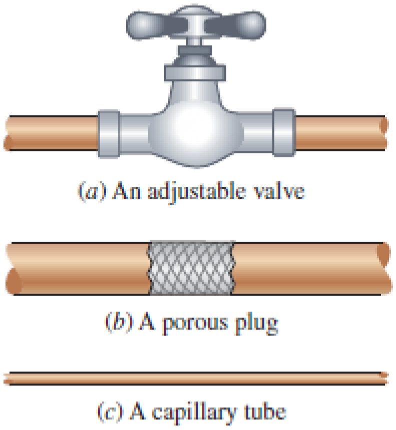


Figure (33) Throttling devices are devices that cause large pressure drops in the fluid.

Throttling devices ⇒ $\boxed{W=0}$ and $\boxed{Q \cong 0}$ and $\boxed{\Delta pe \cong 0}$ and $\boxed{\Delta ke \cong 0}$

Then the conservation of energy equation for this single-stream steady-flow device reduces to:

$$\boxed{h_1 \cong h_2}$$

Example: Air at **(10 °C)**, **(80 kPa)** and enthalpy **(283.14 kJ/kg)** enters the diffuser of a jet engine steadily with a velocity of **(200 m/s)**. The inlet area of the diffuser is (0.4 m^2). The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. The gas constant of air is **(287 J/kg.K)**. **Determine (a) the mass flow rate of the air and (b) the enthalpy of the air leaving the diffuser.**

Solution:

Air is an ideal gas

P_1= 80 KPa $\Rightarrow$ P_1= 80 ×1000= 80000 Pa

T_1= 10 °C $\Rightarrow$ T_1= 10+273.15 = 283.15 K

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$P_1 v_1 = RT_1 \Rightarrow v_1 = \frac{RT_1}{P_1} \Rightarrow v_1 = \frac{287 \times 283.15}{80000} = 1.0158 \text{ m}^3/\text{kg}$$

$$\dot{m}_1 = \rho_1 \dot{V}_1 \Rightarrow \dot{m}_1 = \rho_1 A_1 V_1 \Rightarrow \dot{m}_1 = \frac{1}{v_1} A_1 V_1$$

$$\dot{m}_1 = \frac{1}{1.0158}(0.4)(200) = 78.756 \text{ kg/s}$$

$$\dot{E}_{in} - \dot{E}_{out} = \overbrace{dE/dt}^{0\ \text{(steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

Through a diffuser: $\boxed{\dot{Q} \cong 0}$, $\boxed{\dot{W} = 0}$, $\boxed{\Delta pe \cong 0}$

$$\cancel{\dot{Q}_1} + \cancel{\dot{W}_1} + \dot{m}_1\left(h_1 + \frac{V_1^2}{2} + \cancel{gz_1}\right) = \cancel{\dot{Q}_2} + \cancel{\dot{W}_2} + \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + \cancel{gz_2}\right)$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \Longrightarrow h_2 = h_1 + \frac{V_1^2}{2} - \frac{V_2^2}{2} \Longrightarrow h_2 = h_1 + \left(\frac{V_1^2 - V_2^2}{2}\right)$$

The exit velocity of a diffuser is usually small compared with the inlet velocity ($V_2 << V_1$) thus, the kinetic energy at the exit can be neglected.

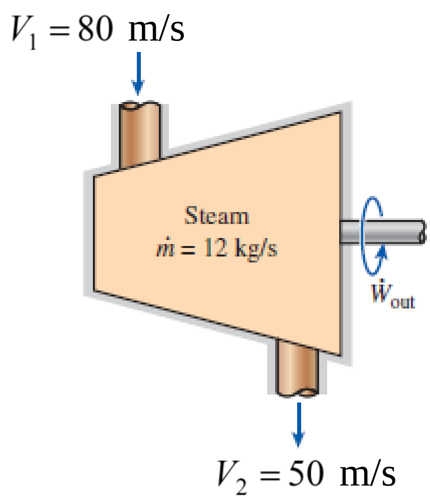
$$h_2 = 283140 + \left(\frac{(200)^2 - 0}{2}\right)$$

$$h_2 = 283140 + \frac{40000}{2}$$

$$h_2 = 303140 \text{ J/kg}$$

$$\boxed{h_2 = 303.14 \text{ kJ/kg}}$$

Example: Steam flows steadily through an adiabatic turbine. The inlet conditions of the steam are velocity **(80 m/s)**, and enthalpy **(3446 kJ/kg)** and the exit conditions are velocity **(50 m/s)**, and enthalpy **(2437.7 kJ/kg)**. The mass flow rate of the steam is **(12 kg/s)**.
Determine:
A. The change in kinetic energy.
B. The power output.



Solution:

A. *The change in kinetic energy is determined from*:

$$\Delta ke = \frac{V_2^{\,2} - V_1^{\,2}}{2} \Longrightarrow \Delta ke = \frac{(50)^2 - (80)^2}{2}$$

$$\Delta ke = \frac{-3900}{2} = -1950 \text{ J/kg} \Longrightarrow \Delta ke = -1.950 \text{ kJ/kg}$$

B.

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{E}_{in} - \dot{E}_{out} = \overbrace{dE/dt}^{0 \text{ (steady)}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

Through turbine: $\boxed{\dot{Q} \cong 0}$ and $\boxed{\Delta pe \cong 0}$

$$\cancel{\dot{Q}_1} + \cancel{\dot{W}_1} + \dot{m}_1\left(h_1 + \frac{V_1^{\,2}}{2} + \cancel{gz_1}\right) = \cancel{\dot{Q}_2} + \dot{W}_2 + \dot{m}_2\left(h_2 + \frac{V_2^{\,2}}{2} + \cancel{gz_2}\right)$$

$$\dot{m}\left(h_1+\frac{V_1^2}{2}\right)=\dot{W}_{out}+\dot{m}\left(h_2+\frac{V_2^2}{2}\right) \Longrightarrow \dot{W}_{out}=\dot{m}\left(h_1+\frac{V_1^2}{2}\right)-\dot{m}\left(h_2+\frac{V_2^2}{2}\right)$$

$$\dot{W}_{out}=\dot{m}\left(h_1+\frac{V_1^2}{2}-h_2-\frac{V_2^2}{2}\right)$$

$$\dot{W}_{out}=12\left(3446000+\frac{(80)^2}{2}-2437700-\frac{(50)^2}{2}\right)$$

$$\dot{W}_{out}=12(1010250)$$

$$\dot{W}_{out}=12123000\ \text{W}$$

$$\dot{W}_{out}=12123\ \text{KW}$$

H.W: Air at **(100 KPa)** and enthalpy **(280.13 kJ/kg)** is compressed steadily to **(600 KPa)** and enthalpy **(400.98 kJ/kg)**. The mass flow rate of the air is **(0.02 kg/s)**, and a heat loss of **(16 kJ/kg)** occurs during the process. Assuming the changes in kinetic and potential energies are negligible. **Determine the necessary power input to the compressor**.