- **Adiabatic process:**
  An adiabatic process is one in which no heat is transferred to or from the fluid during the process.

For adiabatic process of ideal gas $\Longrightarrow$ $\boxed{PV^{\gamma}=C}$

For adiabatic process $\Longrightarrow$ $\boxed{Q=0}$

Example: Prove that the general law of adiabatic process in closed system expressed as $PV^{\gamma}=C$

$$dQ-dw=du$$

For adiabatic process $\Longrightarrow$ $Q=0$

$0-dW=du$ $\Longrightarrow$ $\boxed{du+dw=0}$

$dw=Pdv$ and $du=C_v dT$

$$C_v dT+Pdv=0$$

For ideal gas $Pv=RT$ $\Longrightarrow$ $P=\dfrac{RT}{v}$

$$C_v dT+RT\frac{dv}{v}=0$$

Dividing both sides by T, *we get*:

$$C_v\frac{dT}{T}+R\frac{dv}{v}=0$$ by integral

$$C_v\ln T+R\ln v=C$$

For ideal gas $Pv=RT$ $\Longrightarrow$ $T=\dfrac{Pv}{R}$

$$C_v\ln\frac{Pv}{R}+R\ln v=C$$

Dividing both sides by C_v, *we get*:

$\ln\dfrac{Pv}{R}+\dfrac{R}{C_v}\ln v=\dfrac{C}{C_v}$ Constant $\Longrightarrow$ $\boxed{\ln\dfrac{Pv}{R}+\dfrac{R}{C_v}\ln v=C}$

$$\boxed{C_p = C_v + R} \Longrightarrow R = C_p - C_v \Longrightarrow R = C_v\left(\frac{C_p}{C_v} - 1\right)$$

$$R = C_v(\gamma - 1) \Longrightarrow \boxed{\frac{R}{C_v} = (\gamma - 1)}$$

$$Where: \quad \gamma = \frac{C_p}{C_v}$$

$$\ln\frac{Pv}{R} + (\gamma - 1)\ln v = C \Longrightarrow \ln\frac{Pv}{R} + \ln v^{(\gamma-1)} = C \Longrightarrow \ln\left(\frac{Pv}{R}v^{(\gamma-1)}\right) = C$$

Constant

$$\frac{Pv}{R}v^{(\gamma-1)} = e^{C} \Longrightarrow \frac{Pv}{R}v^{(\gamma-1)} = C$$

Constant

$$Pv.v^{(\gamma-1)} = RC \Longrightarrow Pv.v^{(\gamma-1)} = C \Longrightarrow Pv.v^{\gamma}v^{-1} = C$$

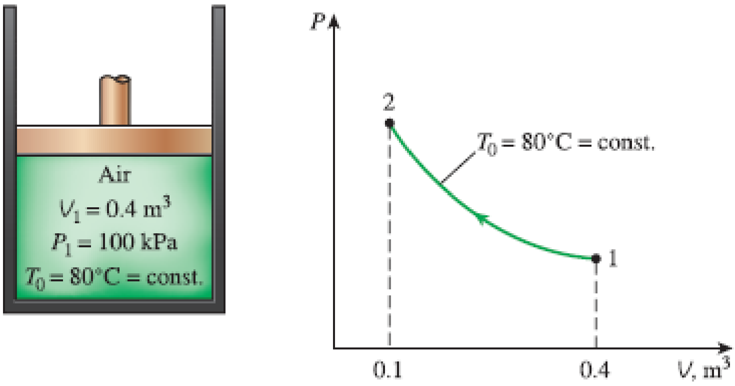
$$PV^{\gamma} = C$$

Relation between P, V, and T:

$$\text{Form equation} \qquad Pv = RT$$

Process	Relation
Isochoric (constant volume) $V_1=V_2$	$\frac{P_1}{P_2}=\frac{T_1}{T_2}$
Isobaric (constant pressure) $P_1=P_2$	$\frac{V_1}{V_2}=\frac{T_1}{T_2}$
Isothermal (constant temperature) $T_1=T_2$	$\frac{P_1}{P_2}=\frac{V_2}{V_1}$
Polytropic $(PV^n=C)$	$\frac{P_2}{P_1}=\left(\frac{V_1}{V_2}\right)^n$ and $\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$
Adiabatic $(PV^\gamma=C)$	$\frac{P_2}{P_1}=\left(\frac{V_1}{V_2}\right)^\gamma$ and $\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

Example: Piston–cylinder device initially contains **(0.4 m^3)** of air at **(100 kPa)** and **(80 ^{o}C)**. The air is now compressed to **(0.1 m^3)** in such a way that the temperature inside the cylinder remains constant, air can be considered to be an ideal gas. **Determine the work done during this process.**



Solution:

$$P_1 = 100 \times 1000 = 100000 \text{ Pa}$$

$$V_1 = 0.4 \text{ m}^3$$

$$V_2 = 0.1 \text{ m}^3$$

At constant temperature $\Longrightarrow$ $\boxed{W = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right)}$

$$W = (100000)(0.4)\ln\left(\frac{0.1}{0.4}\right)$$

$$W = -55451.77 \text{ J}$$

The negative sign indicates that this work is done on the system (a work input), which is always the case for compression processes.

Example: Air undergoes a polytropic compression in a piston–cylinder assembly from **(P_1 =101325 Pa)**, **(T_1 21.11 ^{o}C)** to **(P_2 =506625 Pa)**. Employing the ideal gas model with gas constant **(287 J/kg.K)**, and ($n = 1.3$). **Determine the work per unit mass (J/kg)**.

Solution:

$$P_1 = 101325 \text{ Pa}$$

$$P_2 = 506625 \text{ Pa}$$

$$T_1 = 21.11\,^o\text{C} \Longrightarrow T_1 = 21.11 + 273.15 = 294.26 \text{ K}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Longrightarrow T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Longrightarrow T_2 = 294.26\left(\frac{506625}{101325}\right)^{\frac{1.3-1}{1.3}}$$

$$T_2 = 426.61 \text{ K}$$

$$\boxed{W = \frac{mR(T_2 - T_1)}{1-n}} \Longrightarrow \frac{W}{m} = \frac{R(T_2 - T_1)}{1-n}$$

$$\frac{W}{m} = \frac{287(426.61 - 294.26)}{1 - 1.3}$$

$$\frac{W}{m} = -126614.83 \text{ J/kg}$$

H.W: A piston–cylinder device initially contains **(0.4 kg)** of nitrogen gas at **(160 kPa)** and **(140 ^{o}C)**. The nitrogen is now expanded isothermally to a pressure of **(100 kPa)**. Gas constant for nitrogen (296.8 J/kg.K), and considered nitrogen an ideal gas. **Determine the boundary work done during this process**.

Mass And Energy Analysis of Open System:

Conservation of Mass Principle:

The conservation of mass principle for a control volume can be expressed as: The net mass transfer to or from a control volume is equal to the net change (increase or decrease) of the total mass within the control volume. That is,

Total mass entering the CV	–	Total mass leaving the CV	=	Net change of mass within the CV

$$\boxed{m_{in} - m_{out} = \Delta m_{CV}} \qquad \text{(kg)}$$

Where:

$$\boxed{\Delta m_{CV} = m_{final} - m_{initial}}$$

It can also be expressed in *rate form* as:

$$\boxed{\dot{m}_{in} - \dot{m}_{out} = \Delta m_{CV} / dt} \qquad \text{(kg/s)}$$

Total mass within the CV: $m_{CV} = \int_{CV} \rho dV$

Then the time rate of change of the amount of mass within the control volume is expressed as:

Rate of change of mass within the CV: $\dfrac{dm_{CV}}{dt} = \dfrac{d}{dt}\int_{CV} \rho dV$

Mass balance for steady flow processes:

During **a steady-flow process**, the total amount of mass contained within a control volume does not change with time (m_{CV} =constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it. When dealing with steady-flow processes, we are not interested in the amount of mass that flows in or out of a device over time; instead, we are interested in the amount of mass flowing per unit time, that is, the mass flow rate m. The conservation of mass principle for a general **steady flow** system with multiple inlets and outlets is expressed in rate form as figure (31).

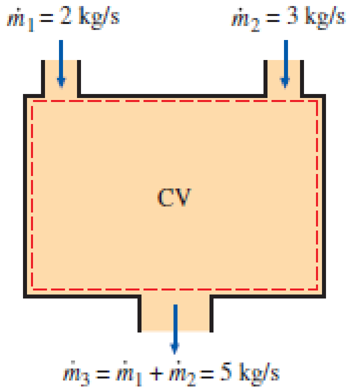


Figure (30) Conservation of mass principle for a two-inlet–one-outlet steady-flow system

Steady flow ⟹

$$\boxed{\sum_{in} \dot{m} = \sum_{out} \dot{m}} \quad \text{(kg/s)}$$

It states that the total rate of mass entering a control volume is equal to the total rate of mass leaving it. Many engineering devices such as **nozzles**, **diffusers**, **turbines**, **compressors**, and **pumps** involve *a single stream* (only one inlet and one outlet). For these cases, we typically denote the inlet state by the subscript **1** and the outlet state by the subscript **2**, and drop the summation signs. Then above equation reduces, for single-stream steady-flow systems, to:

Steady flow (single stream):

$$\boxed{\dot{m}_1 = \dot{m}_2} \Rightarrow \boxed{\rho_1 \dot{V}_1 = \rho_2 \dot{V}_2} \Rightarrow \boxed{\rho_1 V_1 A_1 = \rho_2 V_2 A_2}$$

Where:

$\dot{m}$: mass flow rate (kg/s)

$\dot{V}$: Volume flow rate (m^3/s)

ρ : density (kg/m^3)

A: cross section area