Example: A vessel of volume **(0.2 m^3)** contains nitrogen at **(1.013 bar)** and **(15 ^oC)**. If **(0.2 kg)** of nitrogen is now pumped into the vessel, **calculate the new pressure** when the vessel has returned to its initial temperature. The molecular weight of nitrogen is **(28 kg/kmol)**, and it may be assumed to be ideal gas.

Solution:

$T_1 = 15 + 273.15 = 288.15 \text{ K}$

$P_1 = 1.013 \times 100000 = 101300 \text{ Pa}$

$$R = \frac{R_u}{M} \Rightarrow R = \frac{8314.47}{28} = 296.945 \text{ J/kg.K}$$

$$P_1V_1 = m_1RT_1 \Rightarrow m_1 = \frac{P_1V_1}{RT_1} \Rightarrow m_1 = \frac{101300 \times 0.2}{296.945 \times 288.15}$$

$m_1 = 0.2368 \text{ kg}$

$m_2 = m_1 + 0.2 \Rightarrow m_2 = 0.2368 + 0.2 = 0.4368 \text{ kg}$

$$P_2V_2 = m_2RT_2 \Rightarrow P_2 = \frac{m_2RT_2}{V_2}$$

$V_1 = V_2$ and $T_1 = T_2$

$$P_2 = \frac{0.4368 \times 296.945 \times 288.15}{0.2}$$

$P_2 = 186873.309 \text{ Pa}$

Example: A certain ideal gas of mass **(0.01 kg)** occupies a volume of **(0.003 m^3)** at a pressure of **(7 bar)** and a temperature of **(131 ^oC)**. The gas is allowed to expand until the pressure is **(1 bar)** and the final volume is **(0.02 m^3)**. **Calculate the final temperature**.

Solution:

$m_1 = m_2 = 0.01 \text{ kg}$

$V_1 = 0.003 \text{ m}^3$

$V_2 = 0.02 \text{ m}^3$

$P_1 = 7 \text{ bar} \Rightarrow P_1 = 7 \times 100000 = 700000 \text{ Pa}$

$P_2 = 1 \text{ bar} \Rightarrow P_2 = 1 \times 100000 = 100000 \text{ Pa}$

$T_1 = 131 + 273.15 = 404.15 \text{ K}$

$$P_1V_1 = m_1RT_1 \Rightarrow R = \frac{P_1V_1}{mT_1} \Rightarrow R = \frac{700000 \times 0.003}{0.01 \times 404.15}$$

$$R = \frac{2100}{4.0415} \Rightarrow R = 519.609 \text{ J/kg.K}$$

$$P_2V_2 = m_2RT_2 \Rightarrow T_2 = \frac{P_2V_2}{m_2R_1} \Rightarrow T_2 = \frac{100000 \times 0.02}{0.01 \times 519.609}$$

$$T_2 = \frac{2000}{5.19609} \Rightarrow T_2 = 384.905 \text{ K}$$

H.W: The gage pressure of an automobile tire is measured to be **(210 kPa)** before a trip and **(220 kPa)** after the trip at a location where the atmospheric pressure is **(95 kPa)**. Assuming the volume of the tire remains constant and the air temperature before the trip is **(25 ^oC)**. **Determine air temperature in the tire after the trip.**

Enthalpy: is the sum of the internal energy (U) and the product of pressure (P) and volume (V) .

$$H = U + PV \quad \text{(kJ)}$$

$$h = u + Pv \quad \text{(kJ/kg)}$$

Where:
H: enthalpy
h: specific enthalpy
U: internal energy
P: pressure
V: volume

Specific Heats:

The specific heat is defined as the energy required to raise the temperature of a unit mass of a substance by one degree.

In general, this energy depends on how the process is executed.
In thermodynamics, we are interested in two kinds of specific heats:

- Specific heat at **constant volume** (C_v)
- Specific heat at **constant pressure** (C_p).

Physically, the specific heat at constant volume C_v *can be viewed as the energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant. The energy required to do the same as the pressure is maintained constant is the specific heat at constant pressure* C_p.

The specific heat at constant pressure cp is always greater than cv because at constant pressure the system is allowed to expand and the energy for this expansion work must also be supplied to the system.

$$Cv = \left(\frac{\partial u}{\partial T}\right)_v \qquad C_p = \left(\frac{\partial h}{\partial T}\right)_P$$

$$dQ = mC_p dT \Longrightarrow Q = mC_p(T_2 - T_1) \quad \text{at constant pressure}$$

$$dQ = mC_v dT \Longrightarrow Q = mC_v(T_2 - T_1) \quad \text{at constant volume}$$

Joule's Law:

It has been demonstrated mathematically and experimentally (Joule, 1843) that for an ideal gas the internal energy is a function of the temperature only. That is:

$$u = u(T)$$

(**Joule later showed that for gases that deviate significantly from ideal gas behavior, the internal energy is not a function of temperature alone**.)

Using the definition of enthalpy and the equation of state of an ideal gas, we have:

$$h = u + Pv \quad \text{and} \quad Pv = RT$$
$$h = u + RT$$

Since R is constant and $u = u(T)$, it follows that the enthalpy of an ideal gas is also a function of temperature only:

$$h = h(T)$$

Since u and h depend only on temperature for an ideal gas, the specific heats C_v and C_p also depend, at most, on temperature only. Therefore, at a given temperature (u, h, C_v, and C_p)of an ideal gas have fixed values regardless of the specific volume or pressure.

$$u = u(T)$$
$$h = h(T)$$
$$C_v = C_v(T)$$
$$C_p = C_p(T)$$

Thus, **for ideal gases:**

$$Cv = \left(\frac{\partial u}{\partial T}\right)_v \Longrightarrow du = C_v(T)dT$$

$$C_p = \left(\frac{\partial h}{\partial T}\right)_P \Longrightarrow dh = C_p(T)dT$$

The change in internal energy or enthalpy for an ideal gas during a process from state 1 to state 2 is determined by integrating these equations:

$$\Delta u = u_2 - u_1 = \int_1^2 C_v(T)dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 C_p(T)dT$$

Specific heat relation of ideal gases:

A special relationship between cp and cv for ideal gases can be obtained

$h = u + Pv$ and $Pv = RT$

$h = u + RT$

$dh = du + RdT$

from:

$du = C_v(T)dT$ and $dh = C_p(T)dT$

$C_p dT = C_v dT + RdT$

dividing the resulting expression by dT *, we obtain*:

$$C_p = C_v + R \quad \text{(kJ/kg.K)}$$

Specific heat ratio:

At this point, we introduce another ideal-gas property called the specific heat ratio γ, defined as:

$$\gamma = \frac{C_p}{C_v}$$

The specific ratio also varies with temperature, but this variation is very mild. For monatomic gases, its value is essentially constant at **(1.667)**. Many diatomic gases, including air, have a specific heat ratio of about **(1.4)** at room temperature.

Closed System Process:

- **Isobaric process (constant pressure):**

$P_1 = P_2 = P$

$$W = \int_1^2 PdV \quad \Longrightarrow \quad W = P(V_2 - V_1)$$

From:

$$Q - W = U_2 - U_1$$

Then:

$$Q - (PV_2 - PV_1) = U_2 - U_1 \quad \Longrightarrow \quad Q - PV_2 + PV_1 = U_2 - U_1$$

$$Q = U_2 - U_1 + PV_2 - PV_1 \quad \Longrightarrow \quad Q = (U_2 + PV_2) - (U_1 + PV_1)$$

$$Q = H_2 - H_1$$

$$Q = mC_p(T_2 - T_1)$$

- **Isochoric process (constant volume):**

$\mathbf{V_1 = V_2 = V}$

$$W = \int_1^2 PdV \quad \Longrightarrow \quad W = 0$$

From:

$$Q - W = U_2 - U_1$$

$$Q = U_2 - U_1$$

$$Q = mC_v(T_2 - T_1)$$

- **Isothermal process:**

$$PV = mRT \quad \Longrightarrow \quad \mathbf{T_1 = T_2 = T} \quad \Longrightarrow \quad PV = C \quad \Longrightarrow \quad P = \frac{C}{V}$$

$$W = \int_1^2 PdV \quad \Longrightarrow \quad W = \int_1^2 \frac{C}{V} dV \quad \Longrightarrow \quad W = C\int_1^2 \frac{dV}{V}$$

$$W = C[\ln V]_{V1}^{V2} \quad \Longrightarrow \quad W = C(\ln V_2 - \ln V) \quad \Longrightarrow \quad W = C \ln \frac{V_2}{V_1}$$

$$W = P_1 V_1 \ln \frac{V_2}{V_1}$$

From:

$$Q - W = U_2 - U_1$$

For isothermal process (constant temperature) $U_2 - U_1 = 0$

Then: $Q = W \quad \Longrightarrow \quad Q = W = P_1 V_1 \ln \frac{V_2}{V_1}$

- **Polytropic process:**

During actual expansion and compression processes of gases, pressure and volume are often related by:

$$PV^n = C$$

where (n and C are constants). A process of this kind is called **a polytropic process,** as shown in figure (30).

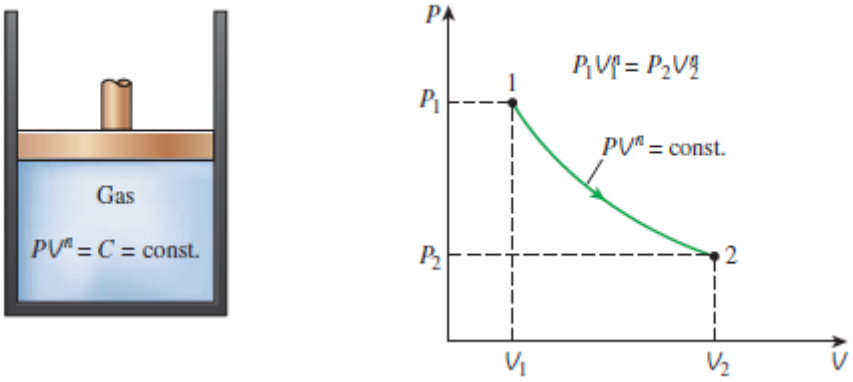

Figure (30) Schematic and P-V diagram for a polytropic process.

Below we develop a general expression for the work done during a polytropic process. The pressure for a polytropic process can be expressed as:

$$P = CV^{-n}$$

$$W = \int_1^2 P\,dV \Longrightarrow W = \int_1^2 CV^{-n}\,dV \Longrightarrow W = C\left[\frac{V^{-n+1}}{-n+1}\right]_{V_1}^{V_2}$$

$$W = \left[\frac{CV_2^{-n+1} - CV_1^{-n+1}}{-n+1}\right] \Longrightarrow W = \left[\frac{CV_2^{-n}V_2 - CV_1^{-n}V_1}{-n+1}\right]$$

Since : $P = CV^{-n}$

Then:

$$W = \left[\frac{P_2V_2 - P_1V_1}{1-n}\right] \quad (n \neq 1)$$

For an ideal gas $\Longrightarrow$ $PV = mRT$

This equation can also be written as:

$$W = \frac{mR(T_2 - T_1)}{1-n} \quad (n \neq 1)$$